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On the Positional and Cinematic Analysis of a Mechanism with Three Independent Contours

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Abstract

In the paper some results concerning the positional and cinematic analysis of a mechanism with three independent contours are presented. The analysis has been done with the method of the projection of the independent contours. Special attention was given to the kinematics of the two component plungers. Finally, some interesting simulation results regarding the influence of the length of the motor crank on the variation of the acceleration of the two plungers are presented.

Key words: *mechanism, kinematics, independent contour*

Introduction

An optimum design of the mechanisms requires as a mandatory step a rigorous analysis of their kinematics behavior [1]. In this step the rod curves for different points on the component links and the values of the displacements of the plungers are established. Also, the variations curves of different cinematic parameters (the speed and the acceleration of different points on the mechanism and the angular speed and acceleration of the links) are determined [2,3].

In this paper some results concerning the positional and cinematic analysis of a mechanism with three independent contours are presented (fig. 1). The analysis has been done with the method of the projection of the independent contours [1]. Special attention was given to the kinematics of the two component plungers. The positional and cinematic parameters were obtained in an analytical form, depending on the crank angle φ_1. For this purpose, a simulation program using Maple programming language [4], that has powerful symbolic computation functions, was developed. Finally, some interesting simulation results regarding the influence of the length of the motor crank on the variation of the acceleration of the two plungers are presented.

Theoretical Considerations and Simulation Results

In Figure 1 the cinematic scheme of a mechanism with three independent contours is presented. As can be seen from the graph associated to the mechanism (fig. 2) the three independent contours are: 0-1-2-3-0 (*O-A-C-O*); 0-1-2-4-5-0 (*O-A-B-D-E-O*) and 0-5-6-7-0 (*E-F-G-E*).

The dimensions of the component links are: $OA = 0.14\,\text{m}$; $AC = 1.05\,\text{m}$; $AB = 0.525\text{m}$; BD=0.25 m; DE=EF=0.225 m; $FG = 1\,\text{m}$; $x_E = 0.5\,\text{m}$; $y_E = -0.3\,\text{m}$. The angular speed of the motor crank *1* of the mechanism is $\omega_1 = 10\,\text{rad/s}$.

The method of the projection of the independent contours [1] has been used for the positional and cinematic analysis. The three independent contours mentioned above have been projected on the *x* and *y* axes (fig. 1) and the following systems of equations were obtained:

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 - s_3 = 0 \\ l_1 \cdot \sin\varphi_1 + l_2 \cdot \sin\varphi_2 = 0 \end{cases} \tag{1}$$

$$\begin{cases} l_1 \cdot \cos\varphi_1 + l_{21} \cdot \cos\varphi_2 + l_4 \cdot \cos\varphi_4 + l_{51} \cdot \cos\varphi_5 - x_E = 0 \\ l_1 \cdot \sin\varphi_1 + l_{21} \cdot \sin\varphi_2 + l_4 \cdot \sin\varphi_4 + l_{51} \cdot \sin\varphi_5 + |y_E| = 0 \end{cases} \tag{2}$$

$$\begin{cases} l_{52} \cdot \cos\varphi_5 + l_6 \cdot \cos\varphi_6 - s_7 = 0 \\ l_{52} \cdot \sin\varphi_5 + l_6 \cdot \sin\varphi_6 = 0 \end{cases} \tag{3}$$

where: $l_1 = OA$; $l_2 = AC$; $l_{21} = AB$; $l_4 = BD$; $l_{51} = DE$; $l_{52} = EF$; $l_6 = FG$; $s_3 = OC$; $s_7 = EG$.

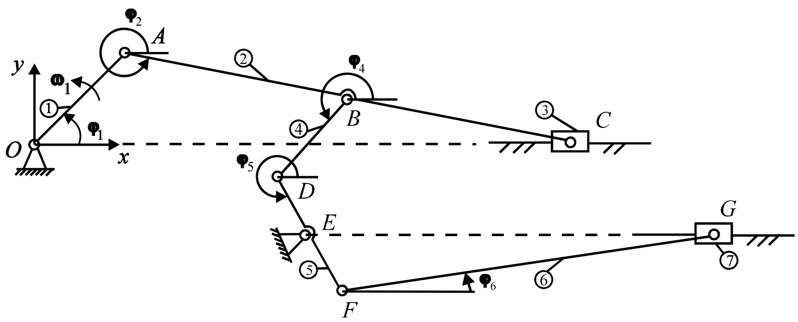

Fig. 1. Mechanism with three independent contours

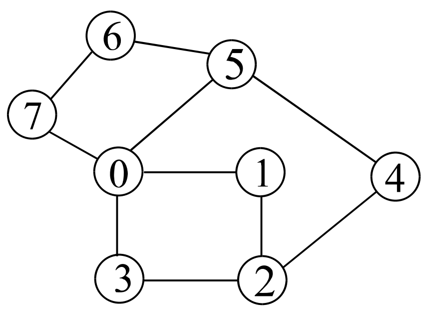

Fig. 2. The graph associated to the mechanism in figure 1

By solving the system of equations (1), the unknown parameters: φ_2 and s_3 can be calculated from the following relations:

$$\begin{cases} \sin\varphi_2 = -\dfrac{l_1}{l_2} \cdot \sin\varphi_1 \\ s_3 = l_1 \cdot \cos\varphi_1 + l_2 \cdot \cos\varphi_2 \end{cases} \tag{4}$$

Then, by solving the systems of equations (2) and (3), the unknown parameters: $\varphi_4, \varphi_5, \varphi_6$ and s_7 can be calculated from the following relations:

$$\begin{cases} A_4 \cdot \cos\varphi_4 + B_4 \cdot \sin\varphi_4 = C_4 \\ A_5 \cdot \cos\varphi_5 + B_5 \cdot \sin\varphi_5 = C_5 \end{cases} \tag{5}$$

$$\begin{cases} \sin\varphi_6 = -\dfrac{l_{52}}{l_6} \cdot \sin\varphi_5 \\ s_7 = l_{52} \cdot \cos\varphi_5 + l_6 \cdot \cos\varphi_6 \end{cases} \tag{6}$$

where:

$$\begin{cases} A_4 = 2 \cdot l_1 \cdot l_4 \cdot \cos\varphi_1 + 2 \cdot l_{21} \cdot l_4 \cdot \cos\varphi_2 - 2 \cdot x_E \cdot l_4 \\ B_4 = 2 \cdot l_1 \cdot l_4 \cdot \sin\varphi_1 + 2 \cdot l_{21} \cdot l_4 \cdot \sin\varphi_2 + 2 \cdot |y_E| \cdot l_4 \\ C_4 = l_{51}^2 - l_1^2 - l_{21}^2 - l_4^2 - x_E^2 - y_E^2 + 2 \cdot x_E \cdot (l_1 \cdot \cos\varphi_1 + l_{21} \cdot \cos\varphi_2) - \\ -2 \cdot l_1 \cdot l_{21} \cdot \cos\varphi_1 \cdot \cos\varphi_2 - 2 \cdot |y_E| \cdot (l_1 \cdot \sin\varphi_1 + l_{21} \cdot \sin\varphi_2) - 2 \cdot l_1 \cdot l_{21} \cdot \sin\varphi_1 \cdot \sin\varphi_2 \end{cases} \tag{7}$$

$$\begin{cases} A_5 = 2 \cdot l_1 \cdot l_{51} \cdot \cos\varphi_1 + 2 \cdot l_{21} \cdot l_{51} \cdot \cos\varphi_2 - 2 \cdot x_E \cdot l_{51} \\ B_5 = 2 \cdot l_1 \cdot l_{51} \cdot \sin\varphi_1 + 2 \cdot l_{21} \cdot l_{51} \cdot \sin\varphi_2 + 2 \cdot |y_E| \cdot l_{51} \\ C_5 = l_4^2 - l_1^2 - l_{21}^2 - l_{51}^2 - x_E^2 - y_E^2 + 2 \cdot x_E \cdot (l_1 \cdot \cos\varphi_1 + l_{21} \cdot \cos\varphi_2) - \\ -2 \cdot l_1 \cdot l_{21} \cdot \cos\varphi_1 \cdot \cos\varphi_2 - 2 \cdot |y_E| \cdot (l_1 \cdot \sin\varphi_1 + l_{21} \cdot \sin\varphi_2) - 2 \cdot l_1 \cdot l_{21} \cdot \sin\varphi_1 \cdot \sin\varphi_2 \end{cases} \tag{8}$$

The relations above have been transposed into a computer program using Maple programming language [4]. The angular and linear speeds and accelerations have been determined by deriving with time the variation functions of the corresponding position parameters determined with the relations (4), (5) and (6). For obtaining the analytical expressions of the speeds and the accelerations mentioned above, the derivatives with respect to the crank angle φ_1 of the position parameters have been calculated using the derivation function *diff* in Maple programming language [4].

Special attention was given to the kinematics of the two component plungers *3* and *7*. In Figures 3 and 4 some simulation results regarding the influence of the length of the motor crank *1* on the variation of the acceleration of the two plungers are presented. In these two figures the variation curves of the acceleration of the two plungers on a cinematic cycle have been obtained by considering that the length l_1 of the motor crank varies between: 0.12 m and 0.2 m.

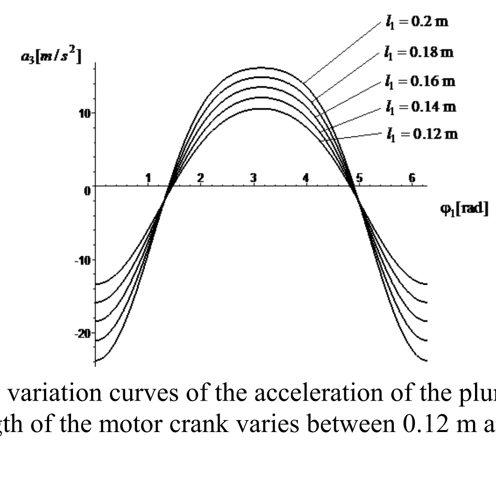

Fig. 3. The variation curves of the acceleration of the plunger *3* when the length of the motor crank varies between 0.12 m and 0.2 m

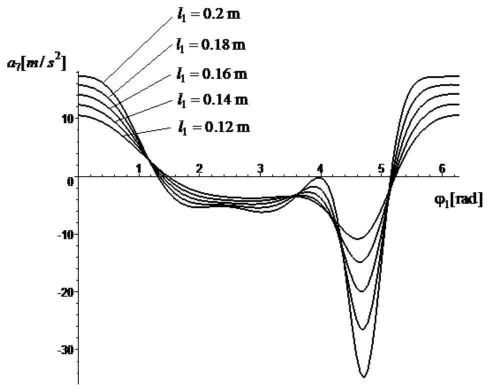

Fig. 4. The variation curves of the acceleration of the plunger *7* when the length of the motor crank varies between 0.12 m and 0.2 m

Conclusions

In this paper some results concerning the positional and cinematic analysis of a mechanism with three independent contours are presented. The analysis has been done with the method of the projection of the independent contours. Special attention was given to the kinematics of the two component plungers. Figures 3 and 4 show that the extreme values of the accelerations of the plungers *3* and *7* have a significant increase when the length of the motor crank increases. So, it is appropriate to reduce the length of the motor crank to a value for which the values of the speeds of the two plungers have acceptable values.

References

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Asupra analizei poziţionale şi cinematice a unui mecanism cu trei contururi independente

Rezumat

In articol sunt prezentate o serie de rezultate privind analiza poziţională şi cinematică a unui mecanism cu trei contururi independente. Analiza s-a realizat folosind metoda proiecţiei contururilor independente. O atenţie deosebită a fost acordată cinematicii mişcării celor două pistoane componente. In final, sunt prezentate o serie de rezultate interesante privind influenţa lungimii manivelei conducătoare asupra variaţiei acceleraţiei celor două pistoane.