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NEW NUMBER NUSSELT CORRELATION FOR THE ANNULAR SPACE OF THE TUBE IN TUBE HEAT EXCHANGER

Popa Maria [1]

[1] Petroleum-Gas University of Ploiesti, Romania
email: mpopa@upg-ploiesti.ro

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ABSTRACT

The use of Nusselt number correlation in the annular space of a tube-in-tube heat exchanger is a difficult task for researchers, since this correlation is deduced specifically for certain geometrical features. As such, researchers would usually only present limited areas of applicability.

This paper presents the algorithm for establishing a new Nusselt number correlation for the annular space of a tube-in-tube heat exchanger with warm water flowing inside in counterflow with cold water in the annular space. The objective of the research was to establish the Nusselt number that allows the calculation of the partial convective heat transfer coefficient on the cold fluid side for a laminar flow regime.

This correlation must be of the form: $Nu = C \cdot Re^m \cdot Pr^n$. This was possible by processing the experimental data by applying the least squares method, as an optimization method. Five sets of experimental data were used for this study. A steel tube in tube heat exchanger was used for this work.

Keywords: heat exchanger, laminar flow regime, partial heat transfer coefficient, Nusselt number

INTRODUCTION

Literature data recommend a wide range of criterial relationship specific to the forced convection heat transfer mechanism, but for limited ranges of applicability (values of Pr and Re numbers) [1-6]. When working on new devices with different geometrical characteristics, these criterial relations can no longer be used otherwise he results obtained by applying the old relations do not optimally correlate experimental data. For the laminar flow regime, the literature recommends the criterial relationships applicable to a tube-in-tube heat exchanger, as shown in Table 1.

There are recent concerns about flow cross sections for fluids, when thermal correlations are established [7-11]. These new flow cross sections are of interest for the study of forced convective heat transfer to find new correlations [12-18].

The relations in Table 1 are recommended for fluid flow through circular spaces and have all been tested on the actual data used for the purposes of this paper. The calculations were performed both from the inside of the small tube to the annular space of the heat exchanger, and vice versa. All relations presented in Table 1 have been analysed to find the relation that would algorithmically lead to the same results for the wall temperatures. Thus, when applying the Sieder-Tate relations, we obtained values for the inner and outer wall temperatures of the small tube with no physical significance, which led to the conclusion that these relations cannot be applied to the appropriate working domains for the present system. Aladiev's relation is an exception because it contains the Grashof criterion and the condition RexPr >1800 must be satisfied. This condition could not be realized by both working fluids. It was determined that the only relation that optimally correlates the experimental data is Hanratty's relation. For the tube-in-tube exchanger in which the working fluids were cold water and a warm food oil, this relation could be used successfully [2].

In the present work, the only working fluid that was used in the inner tube and in the annular space, was water. Calculations were able to determine the constant C and the exponents m and n for the proposed relation, so that the experimental data (the five sets of determinations) were optimally correlated. New Nusselt number correlation established on a new apparatus, $Nu = 0.5293 \cdot Re^{0.7717} \cdot Pr^{0.1718}$ will be used in future experimental determinations.

***Table 1**. Recommended criterial relations for laminar forced convection* [19-26]

No.crt.	Recommended relationships	Authors
1.	$Nu = 1.86 \cdot \left(Re \cdot Pr \cdot \frac{d}{L}\right)^{1/3} \cdot (\frac{\mu}{\mu_p})^{0.14}$	Sieder-Tate
2.	$Nu = 0.475 \cdot (Re \cdot Pr)^{1/3} \cdot (\frac{\mu}{\mu_p})^{0.14}$	Sieder-Tate
3.	$Nu = 3.22 \cdot Re^{1/3} \cdot Pr^{1/3} + 0.117 \cdot Re^{0.8} \cdot Pr^{0.4}$	Beek
4.	$Nu = 0.28 \cdot Re^{0.77} \cdot Pr^{0.4}$	Hanratty
5.	$Nu = 0.74 \cdot f \cdot Re^{0.2} \cdot Pr^{0.3} \cdot Gr^{0.1}$	Aladiev

MATERIALS AND METHODS

In this apparatus warm water circulated in the inner tube, and the cold water circulated in the annular space.

The block diagram of the experimental assembly is presented in figure 1.

The cross-section through the exchanger and its geometrical characteristics are highlighted in Figure 2. The apparatus used in the determination of the geometrical characteristics are shown in Table 2.

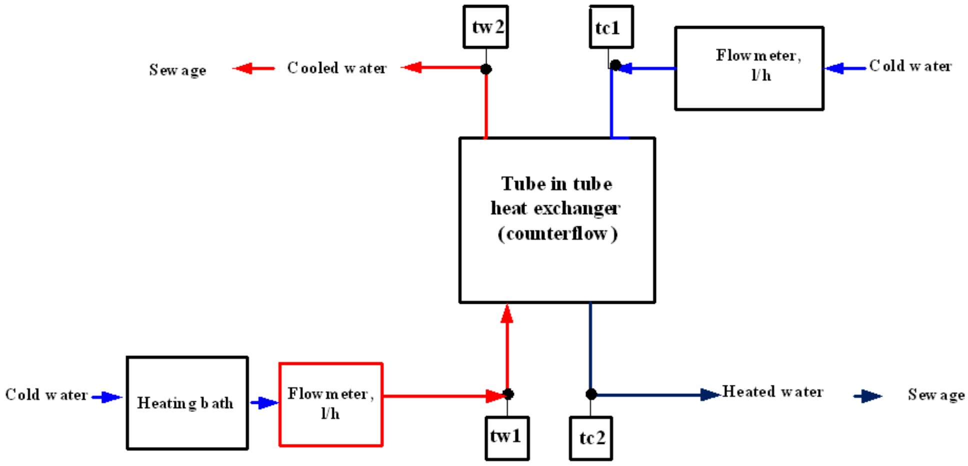

Figure 1. *The block diagram of the experimental assembly*
tc1, tc2 – temperature for cold water (inlet 1, outlet 2),
tw1, tw2 – temperature for warm water (inlet 1, outlet 2)

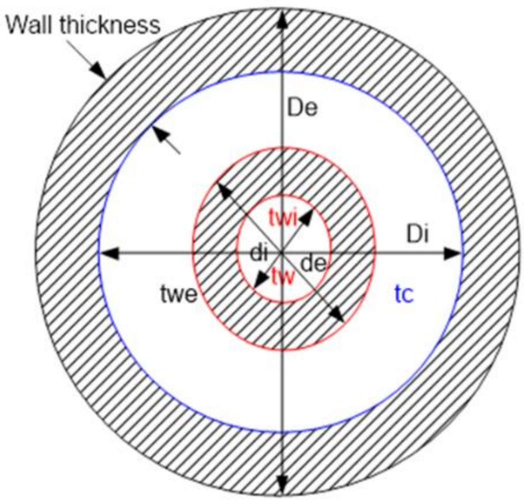

Figure 2. *Geometrical characteristics of the apparatus (De, Di, de, di) and temperature recordings (twe external wall temperature, twi inside wall temperature, tc cold fluid temperature, tw warm fluid temperature)*

Table 2. *Characteristics of the steel heat exchanger*

Geometrical characteristics	**Outer tube**	**Inner tube**
Outside diameter, mm	De = 76	de = 33
Inside diameter, mm	Di = 69	di = 36
Wall thickness, mm	3.5	3.5
Length, m	L = 1	
Thermal conductivity, W/m·^{0}C	40	

Table 3 shows the experimental data for five sets of determinations. These form the basis of the study to establish a new Nu number correlation.

Table 3. *The experimental data*

In the inner tube					
Measured size	Set 1	Set 2	Set 3	Set 4	Set 5
$V_{warm,}$ l/h	60	61	63	69	58
t_{warm1}, °C	71	71	69	68	75
t_{warm2}, °C	50	55	58	59	48
In the annular space					
$V_{cold,}$ l/h	37	25	14	13	50
t_{cold1}, °C	17	17	17	17	17
t_{cold2}, °C	49	54.5	59	62	47

Although the experiments were done on a new heat exchanger, with different dimensions and made of steel, the calculation algorithm used to determine the partial heat transfer coefficients is the one presented in the previous study [2].

All the physical properties of water for this paper were calculated with relations (1)-(4), at mean fluid temperature (t).

These equations are determined graphically, based on literature data [22, 23]:

$$\rho = -0.0036 \cdot t^2 - 0.0697 \cdot t + 1000.5 \tag{1}$$

ρ– density, kg/m^3;

$$c_p = 0.0165 \cdot t^2 - 1.4807 \cdot t + 4205.9 \tag{2}$$

c_p – heat specific, J/kg ·^{0}C;

$$\mu = 2.66 \cdot 10^{-7} \cdot t^2 - 33.82 \cdot 10^{-6} \cdot t + 1.57 \cdot 10^{-3} \tag{3}$$

μ-dynamic viscosity, kg/m·s;

$$\lambda = 10.33 \cdot 10^{-6} \cdot t^2 + 2.33 \cdot 10^{-3} \cdot t + 0.5581 \tag{4}$$

λ-thermal conductivity, W/m·^{0}C.

RESULTS AND DISCUSSIONS

Using the algorithm presented in the previous study [2], keeping the same notations, the obtained results are presented in the tables 3 and 4.

The determination of the wall temperatures were necessary for the calculation of the partial heat transfer coefficient. Thus, the Nusselt number could be calculated from experimental data (Nu_{exp})

Table 3. *Calculated measurements of the warm fluid side*

No. set	$m_w \cdot 10^3$, kg/s	$w_w \cdot 10^3$, m/s	Δt_w, ^0C	Q_t, W	Re_w	Pr_w	Nu_w	α_i, $W/m^2 \cdot {}^0C$	t_{wi}, ^0C
1	16	31.3	21	1437	1613	3.17	131	3303	56
2	17	31.9	24	1112	1646	3.15	133	3356	60
3	17	32.9	11	790	1699	3.14	136	3441	61
4	19	36.1	9	708	1861	3.14	146	3691	62
5	16	30.3	27	1786	1563	3.16	128	3223	56

Table 4. *Calculated measurements of the cold fluid side*

No. set	$m_c \cdot 10^3$, kg/s	Δt_c ^0C	Q_r, W	$w_{cold} \cdot 10^3$, m/s	Re_{cold}	Pr_{cold}	t_{we}, ^0C	α_e, $W/m^2 \cdot {}^0C$	Nu_{exp}
1	10.2	32	1365	3.6	172	5	55	632	37
2	6.9	37.5	1080	2.4	114	4.88	59	466	27
3	3.9	42	677	1.3	76	4.16	60	338	19
4	3.6	45	673	1.2	70	4.21	61	318	18
5	1.4	30	1730	4.8	217	5.39	54	767	45

According to the method of least squares, the linearization of the proposed equation:

$$Nu = C \cdot Re^m \cdot Pr^n \tag{5}$$

will be done by logarithm and the relation becomes:

$$\ln Nu = \ln C + m \ln Re + n \ln Pr \tag{6}$$

All the sizes in the relations (5) and (6) refer only to the cold fluid, the fluid that circulates through the annular space. For the application of the algorithm the following notations are made: $y = ln\ Nu$, $a = ln\ C$, $x = ln\ Re$, $z = ln\ Pr$. According to the method of least squares, the objective function whose minimization is sought is (five number of determinations):

$$F = \sum_{i=1}^{5}[(y_{exp,i} - y_i)^2] = \sum_{i=1}^{5}[\left(y_{exp,i} - a - m \cdot x_i - n \cdot z_i\right)^2] \tag{7}$$

The partial derivatives of this function are:

$$\begin{cases} \frac{\partial F}{\partial a} = \sum_{i=1}^{5}[2 \cdot (y_{exp,i} - a - mx_i - nz_i) \cdot (-1)] \\ \frac{\partial F}{\partial m} = \sum_{i=1}^{5}[2 \cdot (y_{exp,i} - a - mx_i - nz_i) \cdot (-x_i)] \\ \frac{\partial F}{\partial n} = \sum_{i=1}^{5}[2 \cdot (y_{exp,i} - a - mx_i - nz_i) \cdot (-z_i)] \end{cases} \tag{8}$$

The system of linear equations established will be:

$$\begin{cases} \sum_{i=1}^{5} y_i - 5a - m\sum_{i=1}^{5} x_i - n\sum_{i=1}^{5} z_i = 0 \\ \sum_{i=1}^{5} y_i x_i - a\sum_{i=1}^{5} x_i - m\sum_{i=1}^{5} x_i^2 - n\sum_{i=1}^{5} x_i z_i = 0 \\ \sum_{i=1}^{5} y_i z_i - a\sum_{i=1}^{5} z_i - m\sum_{i=1}^{5} x_i z_i - n\sum_{i=1}^{5} z_i^2 = 0 \end{cases} \tag{9}$$

***Table 5**. Results needed to solve the system of equations -1 - based on relation (6) for cold fluid*

No.set	**Re_c**	**Pr_c**	**Nu exp**	**y**	**x**	**z**	**yx**
1	172	5	37	3.611	5.147	1.609	18.587
2	114	4.88	27	3.296	4.736	1.585	15.610
3	76	4.16	19	2.944	4.331	1.426	12.752
4	70	4.21	18	2.891	4.248	1.437	12.280
5	217	5.39	45	3.806	5.380	1.685	20.479
Sum				16.548	23.842	7.742	79.708

***Table 6**. Results needed to solve the system of equations -2- based on relation (6) for cold fluid*

No.set	**x^2**	**xz**	**yz**	**z^2**	**Nu calc**	**RD, %**
1	26.497	8.285	5.812	2.590	36.998	0.1707
2	22.431	7.508	5.224	2.513	27.053	-0.477
3	18.756	6.173	4.197	2.032	18.999	0.631
4	18.050	6.106	4.155	2.066	17.945	-0.105
5	28.943	9.063	6.412	2.838	45.061	-0.179
Sum	114.677	37.135	25.800	12.039	-	-

The relative deviations (RD) calculated with relation (10) are shown in Table 6:

$$RD = \frac{Nu_{calculated} - Nu_{experimental}}{Nu_{experimental}} \cdot 100, \% \tag{10}$$

By replacing the values in tables 5 and 6 you get the system of equations:

$$\begin{cases} 16.548 - 5a - 23.842m - 7.742n = 0 \\ 79.708 - 23.842a - 114.677m - 37.135n = 0 \\ 25.8 - 7.742a - 37.135m - 12.039n = 0 \end{cases} \tag{11}$$

Solving the system of equations (11) led to the values for the sought criterial relationship: a = -0.6361, C = 0.5293, m = 0.7717, n = 0.1718.

The new number Nusselt correlation is:

$$Nu = 0.5293 \cdot Re^{0.7717} \cdot Pr^{0.1718} \tag{12}$$

The values of the Nusselt number calculated with the new relation are given in Table 6.

Future research will reveal new correlations of the Nusselt number, when inside the small tube there will be fixed beds of particles and relation (12) will be used for calculations from the annular space to the inner tube.

Figure 3 shows the variation of the number Nu experimental with the calculated Nu and shows that the points are correctly placed close to the bisector.

Figure 4 shows the variation of the partial heat transfer coefficient for the annular space with cold fluid velocity and shows that all points are optimally correlated.

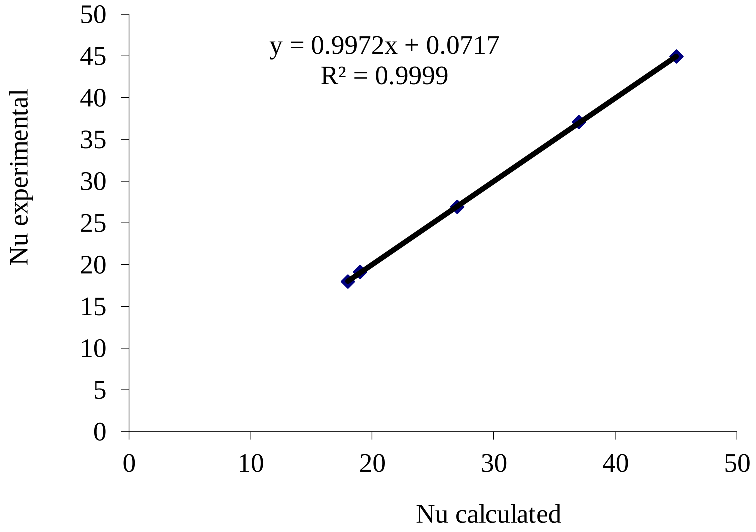

Figure 3. *Variation of experimental Nu number with calculated Nu number for cold fluid*

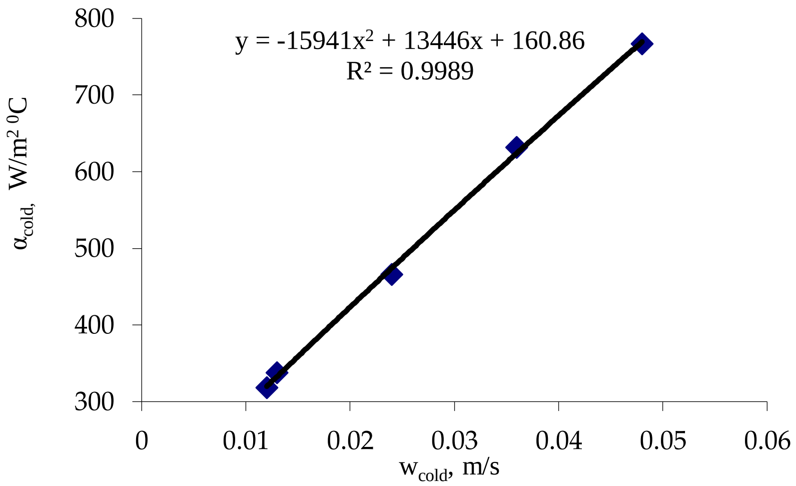

Figure 4. *Variation of partial heat transfer coefficient with cold fluid velocity*

CONCLUSIONS

Based on literature data on heat transfer in cylindrical spaces, it was possible to derive a criterial relationship for the annular space of a tube-in-tube heat exchanger. Determinations were made for warm water in the inner space and for cold water in the annular space.

For the warm fluid, the partial heat transfer coefficients have values in the range 3223-3691 W/m^{20}C at velocities of 0.0303-0.0361m/s, and for the cold fluid, the heat transfer coefficients are in the range 318-767 W/m$^{2}\cdot{}^{0}$C at velocities of 0.012-0.048 m/s. It is important to note that the established new Nusselt number correlation (12) is valid for narrow domains, Pr (4.16-5.39) and Re (70-217).

The five experiments could be satisfactorily correlated in order to establish a new Nusselt number, specific to the geometry of the apparatus shown in the table 2.

The author intends to present the study of heat transfer in a bed of fixed particles in the inner tube, using the new correlation for the annular space.

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