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RESEARCH ON THE OPTIMAL SYNTHESIS OF A MULTI-CONTOUR MECHANISM

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ABSTRACT

The synthesis of mechanisms is a complex field of study that includes, in addition to the knowledge necessary for the correct configuration of working mechanisms and those necessary for their analysis, knowledge from the field of nonlinear programming that involves the calculation of the extrema of an objective function, possibly subject to some constraints. In this context, the dimensional synthesis of the mechanisms that involves the establishment of an optimal set of dimensions of the component elements to ensure the necessary precision of the operating parameters remains an always current field through its multiple applications. The paper presents a method of dimensional synthesis of a multi-contour mechanism for which certain values of the stroke of a component piston and the positions of the driving element of the mechanism in which the change of the direction of movement of the piston takes place are imposed. Through the dimensional synthesis of the analyzed mechanism, it is aimed that, in addition to the mentioned conditions, a minimum value of the sum of the lengths of the component elements is obtained. To achieve the optimal synthesis, the Sequential Quadratic Programming (SQP) method included in the Optimization package from the Maple program was used.

Keywords*:* mechanism, dimensional synthesis, optimization, objective function, Sequential Quadratic Programming

INTRODUCTION

The creation of efficient working mechanisms, which ensure a high precision of the operating parameters often requires, in addition to their analysis, the realization of a dimensional synthesis that leads to the achievement of some imposed optimization criteria [1-11]. The dimensional synthesis of mechanisms includes, in addition to the knowledge necessary for the correct configuration of working mechanisms and those necessary for their analysis, knowledge from the field of nonlinear programming that involves the calculation of the extrema of an objective function, possibly subject to some constraints [12-25].

The scope of the present paper is to develop a method of dimensional synthesis of a multi-contour mechanism for which certain values of the stroke of a component piston and the positions of the driving element of the mechanism in which the change of the direction of movement of the piston takes place are imposed. Through the dimensional synthesis of the analyzed mechanism, it is aimed that, in addition to the mentioned conditions, a minimum value of the sum of the lengths of the component elements is obtained. To achieve the optimal synthesis, the Sequential Quadratic Programming (SQP) method included in the Optimization package from the Maple program was used [26]. The methodology used in the development of the operating simulator of the analyzed mechanism has a novelty character and offers multiple possibilities to include the optimization functions in Maple in the synthesis problems of the mechanisms.

THEORETICAL CONSIDERATIONS AND RESULTS OF THE SIMULATIONS

Figure 1 shows the kinematic diagram of the mechanism whose dimensional synthesis is to be achieved. The mechanism consists of four bar-type elements and a piston. From a structural point of view, the mechanism has two contours in its component: 0-1-2-3-0 and 0-1-2-4-5-0. The closing vector equations of the two contours are as follows:

$$\begin{cases} \overline{O_1A} + \overline{AB} + \overline{BO_3} + \overline{O_3O_1} = 0 \\ \overline{O_1A} + \overline{AC} + \overline{CD} + \overline{DO_1} = 0 \end{cases} \tag{1}$$

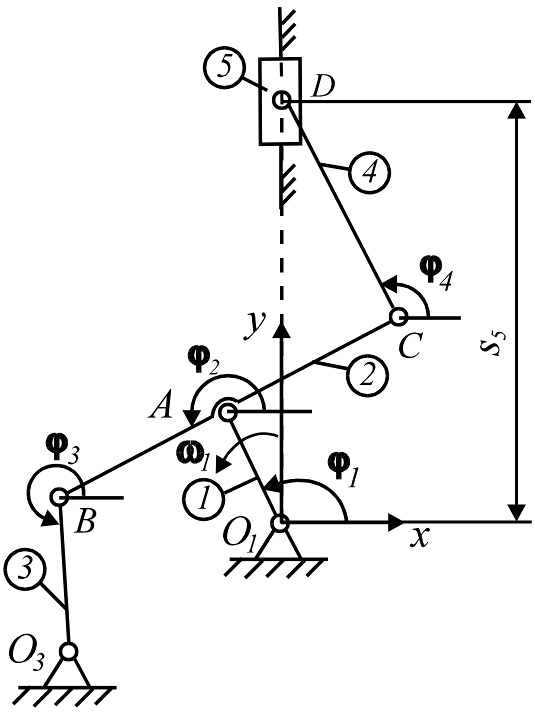

Figure 1. *Plane multi-contour mechanism*

By projecting the two vector equations from (1) on the axes of the (Oxy) coordinate system, the following systems of equations are obtained:

$$\begin{cases} O_1A \cdot \cos\varphi_1 + AB \cdot \cos\varphi_2 + BO_3 \cdot \cos\varphi_3 + \left|x_{O_3}\right| = 0 \\ O_1A \cdot \sin\varphi_1 + AB \cdot \sin\varphi_2 + BO_3 \cdot \sin\varphi_3 + \left|y_{O_3}\right| = 0 \end{cases} \tag{2}$$

$$\begin{cases} O_1A \cdot \cos\varphi_1 + AC \cdot \cos(\varphi_2 - \pi) + CD \cdot \cos\varphi_4 = 0 \\ O_1A \cdot \sin\varphi_1 + AC \cdot \sin(\varphi_2 - \pi) + CD \cdot \sin\varphi_4 - s_5 = 0 \end{cases} \tag{3}$$

By solving the system of equations (2), the angles φ_2 and φ_3 can be determined with the relations:

$$\begin{cases} \varphi_2 = (-1)^k \cdot \arcsin\dfrac{P_2}{\sqrt{M_2^2 + N_2^2}} - ATAN2(M_2, N_2) \\ \varphi_3 = ATAN2(-O_1A \cdot \sin\varphi_1 - AB \cdot \sin\varphi_2 - \left|y_{O_3}\right|, -O_1A \cdot \cos\varphi_1 - AB \cdot \cos\varphi_2 - \left|x_{O_3}\right|) \end{cases} \tag{4}$$

where *ATAN2*(*y*,*x*) calculates arctan(*y/x*) taking into account the signs of the arguments *y* and *x* and:

$$\begin{cases} M_2 = 2 \cdot O_1A \cdot AB \cdot \cos\varphi_1 + 2 \cdot AB \cdot \left|x_{O_3}\right| \\ N_2 = 2 \cdot O_1A \cdot AB \cdot \sin\varphi_1 + 2 \cdot AB \cdot \left|y_{O_3}\right| \\ P_2 = BO_3^2 - O_1A^2 - AB^2 - x_{O_3}^2 - y_{O_3}^2 - 2 \cdot O_1A \cdot \left|x_{O_3}\right| \cdot \cos\varphi_1 - 2 \cdot O_1A \cdot \left|y_{O_3}\right| \cdot \sin\varphi_1 \end{cases} \tag{5}$$

By solving the system of equations (3), the displacement s_5 of the piston *5* and the angle φ_4 can be determined with the relations:

$$\begin{cases} s_5 = R_5 + \sqrt{R_5^2 - T_5} \\ \varphi_4 = ATAN2(-O_1A \cdot \sin\varphi_1 + AC \cdot \sin\varphi_2 + s_5, -O_1A \cdot \cos\varphi_1 + AC \cdot \cos\varphi_2) \end{cases} \tag{6}$$

where:

$$\begin{cases} R_5 = O_1A \cdot \sin\varphi_1 - AC \cdot \sin\varphi_2 \\ T_5 = -CD^2 + O_1A^2 + AC^2 - 2 \cdot O_1A \cdot AC \cdot \cos\varphi_1 \cdot \cos\varphi_2 - 2 \cdot O_1A \cdot AC \cdot \sin\varphi_1 \cdot \sin\varphi_2 \end{cases} \tag{7}$$

The speed and acceleration of piston *5* were determined with the relations:

$$\begin{cases} v_5 = \dfrac{\mathrm{d}s_5}{\mathrm{d}\varphi_1} \cdot \omega_1 \\ a_5 = \dfrac{\mathrm{d}v_5}{\mathrm{d}\varphi_1} \cdot \omega_1 \end{cases} \tag{8}$$

where ω_1 is the angular speed of the driving element *1*.

Based on the presented relationships, a functioning simulator was created using the Maple program. In the realization of the dimensional synthesis of the analyzed mechanism, it started with the following dimensions of the component elements: $O_1A = 0.03\,\mathrm{m}$; $AB = 0.3\,\mathrm{m}$; $BO_3 = 0.25\,\mathrm{m}$; $AC = 0.35\,\mathrm{m}$; $CD = 0.5\,\mathrm{m}$; $x_{O_3} = -0.3\,\mathrm{m}$; $y_{O_3} = -0.2\,\mathrm{m}$. For the angular speed ω_1 of the driving element *1* it was considered the value $10\,\mathrm{rad/s}$.

Figures 2, 3 and 4 show the variation curves on the kinematic cycle for displacement s_5, speed v_5 and acceleration a_5 of the piston in the mechanism component.

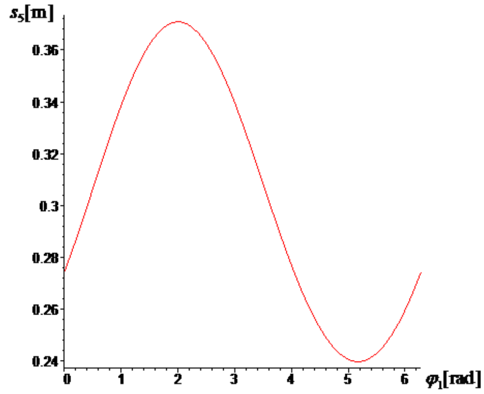

Figure 2. *The variation of displacement* s_5 *for the initial dimensions of the component elements*

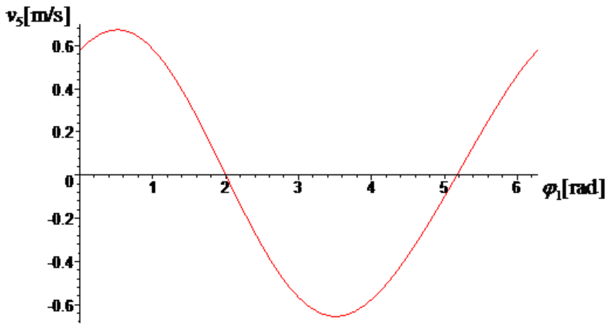

Figure 3. *The variation of speed* v_5 *for the initial dimensions of the component elements*

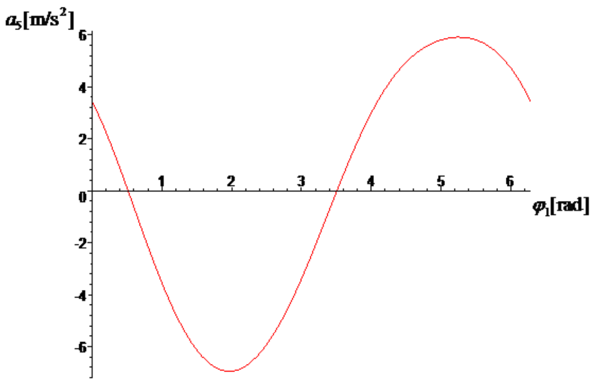

Figure 4. *The variation of acceleration* a_5 *for the initial dimensions of the component elements*

NLPSolve function from the Maple *Optimization* package that can operate with the Sequential Quadratic Programming (SQP) method was used to perform the dimensional synthesis of the mechanism. In the simulations regarding the synthesis of the analyzed mechanism, the value of the stroke of the piston from the component of the mechanism and the value of the crank angle φ_1 for its extreme positions were imposed, in the conditions in which it was aimed to reach a minimum for the sum of the lengths of the bar-type component elements.

Figures 5, 6 and 7 show the variation curves for displacement s_5, speed v_5 and acceleration a_5 obtained when the stroke value of piston 5 is equal to 0.1 m, and the extreme positions of the piston are reached for $\varphi_1 = 2\pi/3$, respectively $\varphi_1 = 5\pi/3$. In this first case where the synthesis of the mechanism was carried out, the following values were obtained for the dimensions of the component elements: $O_1A = 0.0225\ \text{m}$; $AB = 0.1848\ \text{m}$; $BO_3 = 0.125\ \text{m}$; $AC = 0.1783\ \text{m}$; $CD = 0.2758\ \text{m}$; $x_{O_3} = -0.1499\ \text{m}$; $y_{O_3} = -0.1079\ \text{m}$.

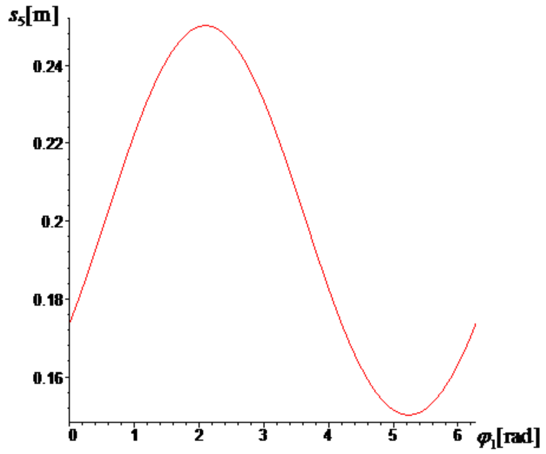

Figure 5. *The variation of displacement* s_5 *for the first analyzed synthesis case*

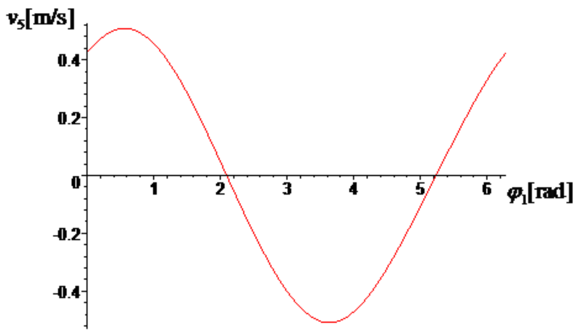

Figure 6. *The variation of speed* v_5 *for the first analyzed synthesis case*

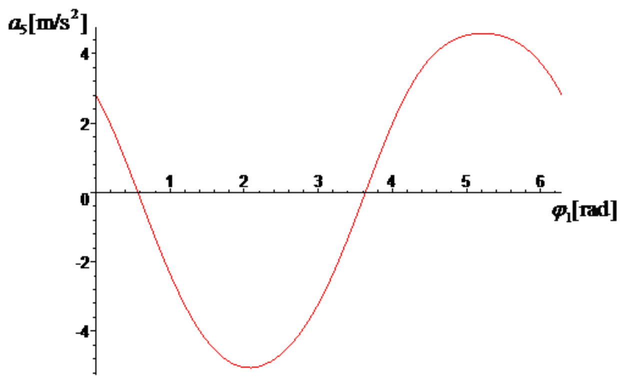

Figure 7. *The variation of acceleration* a_5 *for the first analyzed synthesis case*

Figures 8, 9 and 10 show the variation curves for displacement s_5, speed v_5 and acceleration a_5 obtained when the stroke value of piston 5 is equal to 0.2 m, and the extreme positions of the piston are reached for $\varphi_1 = \pi/2$, respectively $\varphi_1 = 3\pi/2$. In this second case of synthesis, the following values were obtained for the dimensions of the component elements: $O_1A = 0.0449$ m; $AB = 0.1499$ m; $BO_3 = 0.1365$ m; $AC = 0.2153$ m; $CD = 0.5007$ m; $x_{O_3} = -0.1892$ m; $y_{O_3} = -0.1001$ m.

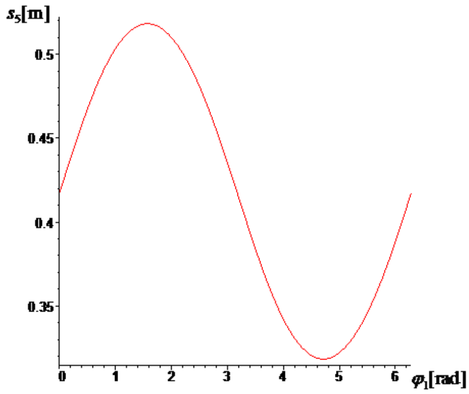

Figure 8. *The variation of displacement s_5 for the second analyzed synthesis case*

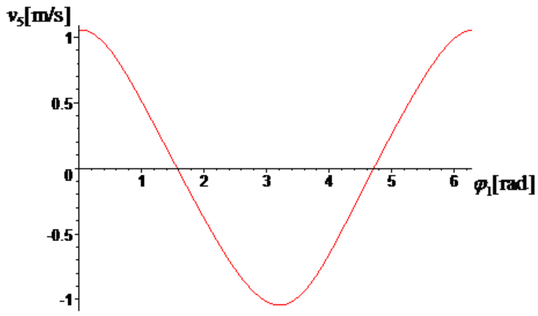

Figure 9. *The variation of speed v_5 for the second analyzed synthesis case*

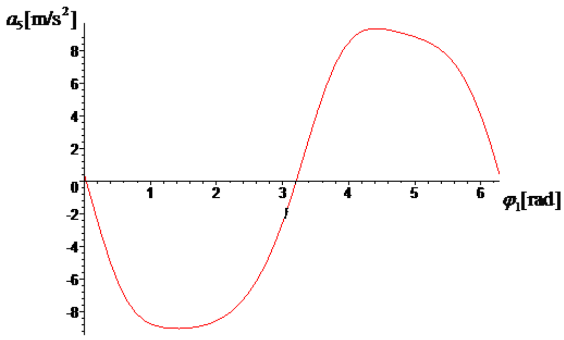

Figure 10. *The variation of acceleration a_5 for the second analyzed synthesis case*

Figures 5÷7 and respectively 8÷10 highlight the fact that the results obtained by the proposed synthesis method are correct both regarding the stroke of the piston and its extreme positions. The variation curves of the speed and acceleration of the piston additionally demonstrate the good functioning of the mechanism in the two analyzed cases of synthesis.

CONCLUSIONS

In this paper has been developed a method of dimensional synthesis of a multi-contour mechanism for which certain values of the stroke of a component piston and the positions of the driving element of the mechanism in which the change of the direction of movement of the piston takes place are imposed. Through the dimensional synthesis of the analyzed mechanism, it has been aimed that, in addition to the working conditions, a minimum value of the sum of the lengths of the component elements was obtained. The positional analysis of the mechanism was carried out on each component contour. Based on this analysis, a functional simulator was created. The simulator allows establishing the variation on the kinematic cycle of all the operating parameters of the analyzed mechanism. The optimal synthesis has been achieved by using the Sequential Quadratic Programming (SQP) method included in NLPSolve function from Optimization package of Maple program. The results of the simulations presented in the paper confirm with their precision the validity of the developed methodology. In the end, it can be concluded that the methodology and results presented in the paper are useful to all those who have concerns in the field of dimensional synthesis of multi-contour mechanisms.

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