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RESEARCH ON THE OPTIMAL PERFORMANCE OF TASKS UNDER REDUNDANCY CONDITIONS IN THE CASE OF A ROBOT WITH FIVE DEGREES OF FREEDOM

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ABSTRACT

The fulfillment of programmed tasks by industrial robots in redundant conditions occurs when a robotic system has more degrees of freedom than would be necessary to accomplish the task. From the point of view of mechanical modeling of robotic structures, this translates into the fact that the number of motor coordinates of the robot is greater than would be necessary and sufficient to ensure the position and orientation of the robot's end effector required by the task. In such situations, there is the possibility of optimally performing the programmed task under the conditions of meeting certain criteria that may refer to better accessibility of the robot's workspace in the presence of obstacles, achieving a minimum in terms of movements at the level of the robot's motor axes or obtaining motor coordinate values as far as possible from the edges of their variation ranges. The paper analyzes the case of a robot with five rotation modules whose task is to ensure a trajectory imposed at the origin of the coordinate system attached to its end effector. The redundancy case is analyzed by imposing as an optimization criterion the achievement of a minimum of displacements at the motor axes level of the analyzed robot. Finally, the results obtained for the variation of motor coordinates in the case of a linear interpolation of the end effector movement are presented.

Keywords: robot, degree of freedom, redundancy, optimal functioning, movement interpolation

INTRODUCTION

The analysis of mechanisms in general and of the active ones in the structure of industrial robots always remains a current subject of study [1-14]. Mechanical modeling of these mechanisms is necessary to ensure proper functioning in view of meeting certain optimization criteria [15-29]. In the practice of using robotic systems in various industrial applications, situations frequently arise in which the robot possesses more degrees of freedom than would be necessary and sufficient to perform a certain task. These situations correspond to cases of redundancy in which there is the possibility that the robot meets certain optimization criteria in performing the programmed task. These criteria may refer to better accessibility of the robot's workspace in the presence of obstacles, achieving a minimum in terms of movements at the level of the robot's motor axes or obtaining motor coordinate values as far as possible from the edges of their variation ranges.

In this paper it is analyzed the case of a robot with five degrees of freedom whose task is to ensure a trajectory imposed at the origin of the coordinate system attached to its end effector. The redundancy case is analyzed by imposing as an optimization criterion the achievement of a minimum of displacements at the motor axes level of the analyzed robot. Finally, the results obtained with a program developed by the authors using Maple [30] present the variation of the motor coordinates in the case of a linear interpolation of the end effector movement. The methodology for generating the inverse geometric model in the redundancy case studied, as well as the simulator developed by the authors has a novelty character, offering the possibility of analyzing also other optimization criteria than the one presented.

MATERIALS AND METHODS

The robotic mechanism presented in figure 1 has five rotation modules. $(O_i x_i y_i z_i), i=\overline{0,5}$, are attached to the modules $\overline{1,5}$ and to the base of the mechanism. The matrices ${}^{i-1}R_i, i=\overline{1,5}$, are as follows:

$$\begin{cases} {}^0R_1 = R(z,q_1) = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \ {}^1R_2 = R(x,q_2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_2 & -\sin q_2 \\ 0 & \sin q_2 & \cos q_2 \end{bmatrix}; \\ {}^2R_3 = R(x,q_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_3 & -\sin q_3 \\ 0 & \sin q_3 & \cos q_3 \end{bmatrix}; \ {}^3R_4 = R(y,q_4) = \begin{bmatrix} \cos q_4 & 0 & \sin q_4 \\ 0 & 1 & 0 \\ -\sin q_4 & 0 & \cos q_4 \end{bmatrix}; \\ {}^4R_5 = R(z,q_5) = \begin{bmatrix} \cos q_5 & -\sin q_5 & 0 \\ \sin q_5 & \cos q_5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{cases} \tag{1}$$

where: q_1, q_2, q_3, q_4, q_5 are the motors coordinates.

The position of the characteristic point O_T (figure 1) relative to $(O_0 x_0 y_0 z_0)$ can be determined as follows:

$$^{(0)}O_0O_T = \begin{bmatrix} ^{(0)}x_{O_T} \\ ^{(0)}y_{O_T} \\ ^{(0)}z_{O_T} \end{bmatrix} = \sum_{i=1}^{4} {}^0R_i \cdot {}^{(i)}O_iO_{i+1} + {}^0R_5 \cdot {}^{(5)}O_5O_T \tag{2}$$

where: $^{(1)}O_1O_2 = [0 \quad 0 \quad l_1]^T$; $^{(2)}O_2O_3 = [0 \quad l_2 \quad 0]^T$; $^{(3)}O_3O_4 = [0 \quad l_3 \quad 0]^T$; $^{(4)}O_4O_5 = [0 \quad l_4 \quad 0]^T$; $^{(5)}O_5O_T = [0 \quad l_5 \quad 0]^T$

${}^0R_2 = {}^0R_1 \cdot {}^1R_2$; ${}^0R_3 = {}^0R_2 \cdot {}^2R_3$; ${}^0R_4 = {}^0R_3 \cdot {}^3R_4$; ${}^0R_5 = {}^0R_4 \cdot {}^4R_5$.

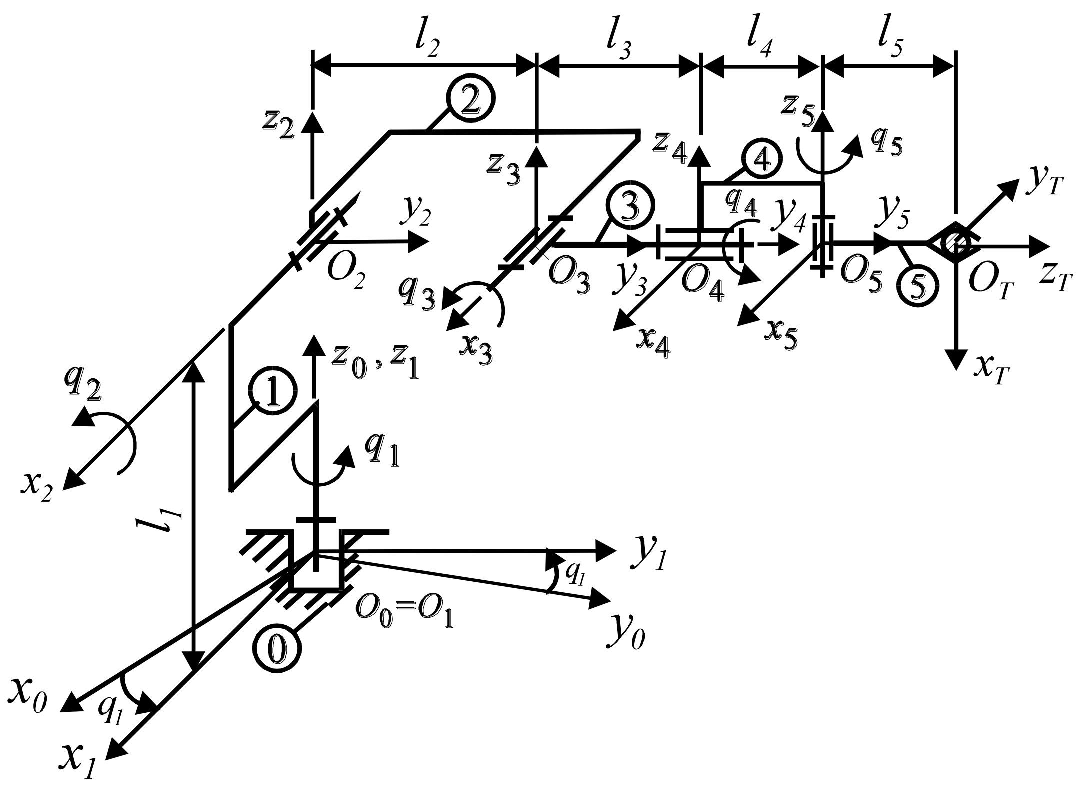

Figure 1. Robotic mechanism with five rotation modules

We considered that a linear movement of the point O_T has to be generated. The trajectory followed was discretized into a number n of intermediate positions, equally spaced between them. The coordinates of O_T in these positions may be determined as follows:

$$\begin{cases} {}^{(0)}x_{O_T}(k) = {}^{(0)}x_{O_{T_i}} + \frac{k}{n} \cdot \left({}^{(0)}x_{O_{T_f}} - {}^{(0)}x_{O_{T_i}} \right) \\ {}^{(0)}y_{O_T}(k) = {}^{(0)}y_{O_{T_i}} + \frac{k}{n} \cdot \left({}^{(0)}y_{O_{T_f}} - {}^{(0)}y_{O_{T_i}} \right) \quad k = \overline{1,n} \\ {}^{(0)}z_{O_T}(k) = {}^{(0)}z_{O_{T_i}} + \frac{k}{n} \cdot \left({}^{(0)}z_{O_{T_f}} - {}^{(0)}z_{O_{T_i}} \right) \end{cases} \tag{3}$$

where: O_{T_i} and O_{T_f} are the initial and the final position, respectively, of the point O_T.

The variation of the motors coordinates, grouped in the vector $q = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5]^T$, has been determined iteratively with the relationship:

$$q(k+1) - q(k) = J^+(q(k)) \cdot \left(O_0O_T(k+1) - O_0O_T(k) \right) \tag{4}$$

where: J^+ is the pseudo-inverse [12] corresponding to the robot's mechanism:

$$J^+ = J^T \cdot \left(J \cdot J^T \right)^{-1} \tag{5}$$

where: J is the jacobean matrix:

$$J=\begin{bmatrix} \frac{d\,^{(0)}x_{O_T}}{dq_1} & \frac{d\,^{(0)}x_{O_T}}{dq_2} & \frac{d\,^{(0)}x_{O_T}}{dq_3} & \frac{d\,^{(0)}x_{O_T}}{dq_4} & \frac{d\,^{(0)}x_{O_T}}{dq_5} \\ \frac{d\,^{(0)}y_{O_T}}{dq_1} & \frac{d\,^{(0)}y_{O_T}}{dq_2} & \frac{d\,^{(0)}y_{O_T}}{dq_3} & \frac{d\,^{(0)}y_{O_T}}{dq_4} & \frac{d\,^{(0)}y_{O_T}}{dq_5} \\ \frac{d\,^{(0)}z_{O_T}}{dq_1} & \frac{d\,^{(0)}z_{O_T}}{dq_2} & \frac{d\,^{(0)}z_{O_T}}{dq_3} & \frac{d\,^{(0)}z_{O_T}}{dq_4} & \frac{d\,^{(0)}z_{O_T}}{dq_5} \end{bmatrix} \tag{6}$$

The values of the motor coordinates for $k=1$ were numerically calculated by solving the system of equations (2) when: $^{(0)}O_0O_T={}^{(0)}O_0O_{T_i}$ and by imposing for the motor coordinates q_4 and q_5 certain values in their range of variation.

Relation (4) allows the determination of the variation of the motor coordinates that ensures the achievement of a minimum of displacements at the motor axes level of the analyzed robot.

RESULTS AND DISCUSSIONS

A simulator has been developed by the authors using Maple [26]. The parameters involved in calculations have been considered with the following values: $l_1=1.5\text{ m}$; $l_2=0.7\text{ m}$; $l_3=0.45\text{ m}$; $l_4=0.35\text{ m}$; $l_5=0.25\text{ m}$;

$^{(0)}O_0O_{T_i}=\begin{bmatrix}0.3 & 0.45 & 0.75\end{bmatrix}^T$; $^{(0)}O_0O_{T_f}=\begin{bmatrix}0.45 & 0.75 & 0.9\end{bmatrix}^T$;

The values obtained for q_1, q_2 and q_3, when: $^{(0)}O_0O_T={}^{(0)}O_0O_{T_i}$, by imposing: $q_4=30^\circ$ and $q_5=60^\circ$, for one of the robot's working configurations, are as follow: $q_1=-0.942\text{ rad}$; $q_2=0.238\text{ rad}$ and $q_3=-2.112\text{ rad}$.

The number n of intermediate positions used in simulations has been considered equal to 50. The obtained variation curves of q_1, q_2, q_3, q_4 and q_5, depending on position(k) (k=1,2,…,n) on the segment traveled by O_T, are presented below.

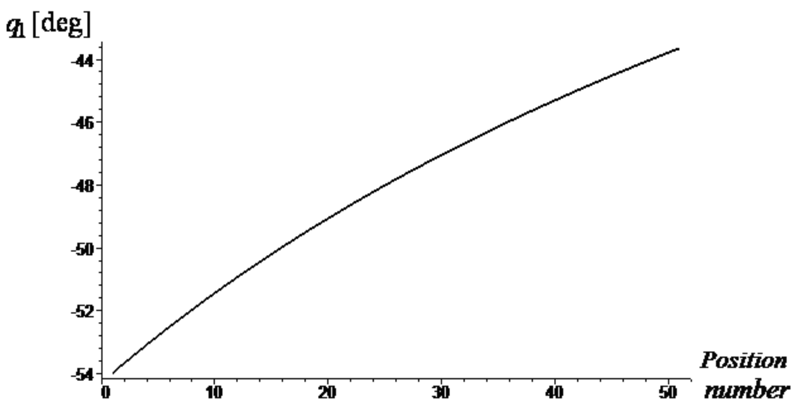

Figure 2. *The variation curve of the motor coordinate* q_1

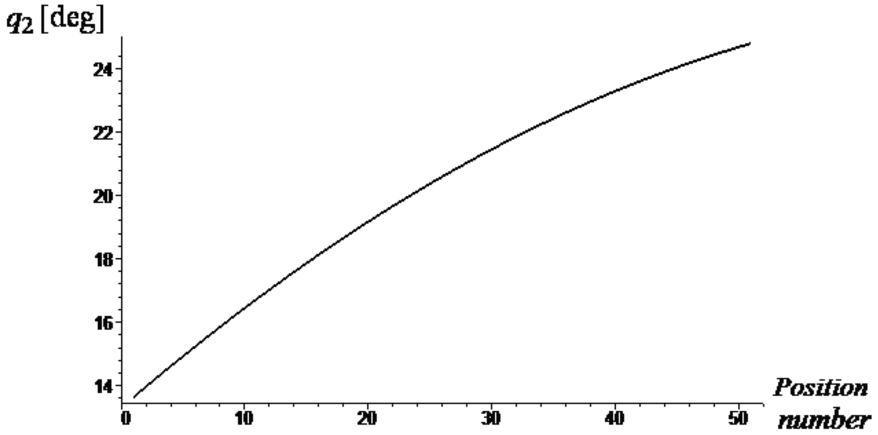

Figure 3. *The variation curve of the motor coordinate* q_2

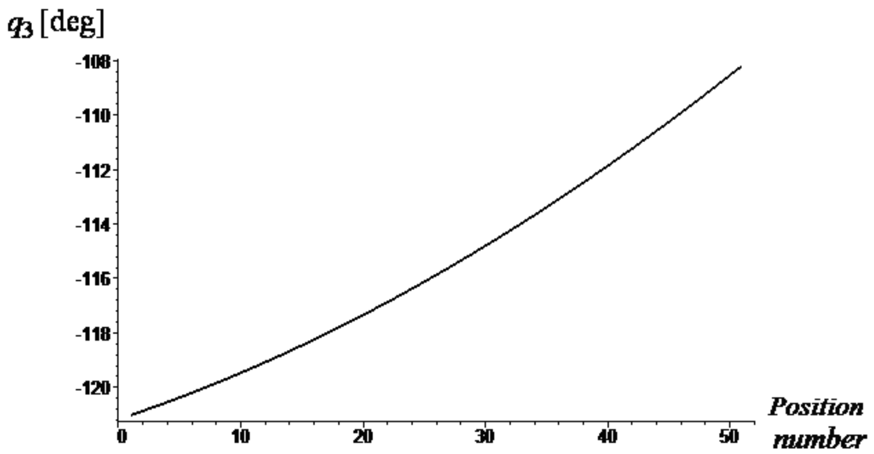

Figure 4. *The variation curve of the motor coordinate* q_3

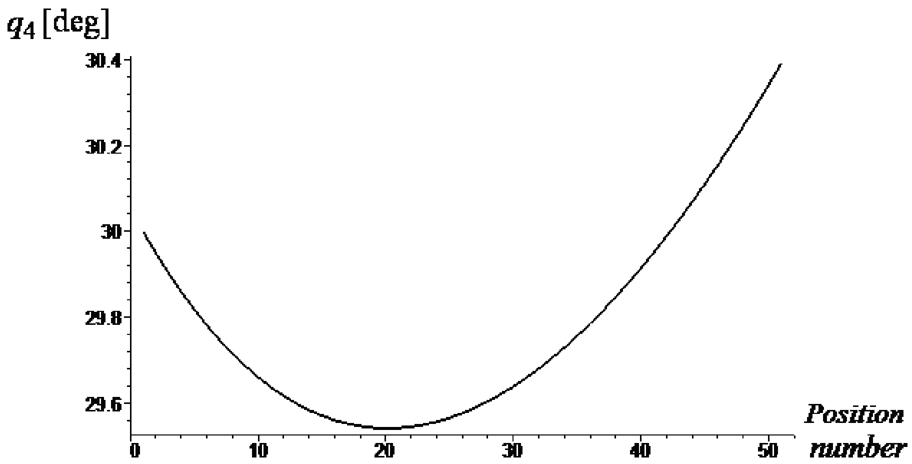

Figure 5. *The variation curve of the motor coordinate* q_4

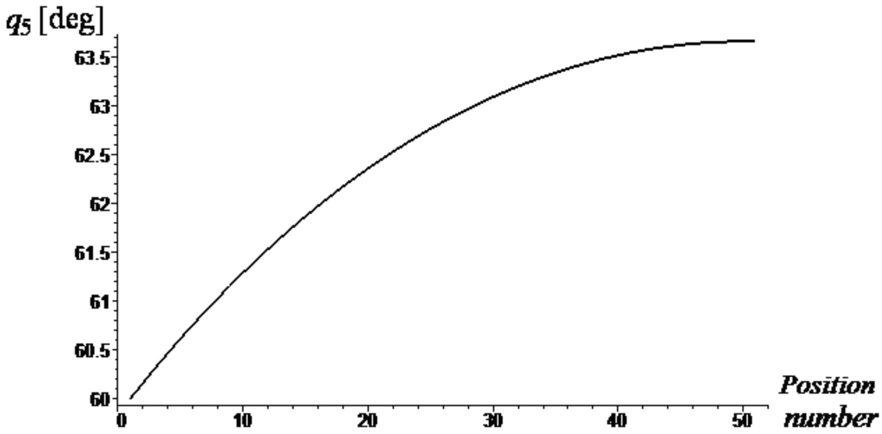

Figure 6. *The variation curve of the motor coordinate* q_5

The variation curves of $q_i, i=\overline{1,5}$, highlight a continuous variation of them, without presenting sudden variations that will lead to improper operation of the robot. The following values for the angular displacements at the motors axes level have been obtained: $\Delta q_1 = 10.328^\circ$; $\Delta q_2 = 11.143^\circ$; $\Delta q_3 = 12.831^\circ$; $\Delta q_4 = 1.310^\circ$ and $\Delta q_5 = 3.661^\circ$.

CONCLUSIONS

Optimizing the functioning of industrial robots in performing different tasks can be related to several aspects that can look at the obstacles in the workspace, minimal energy consumption, avoiding singular configurations during operation, providing positions at the motor axes as far as the race limiters, ensuring a higher manipulability, etc. This problem becomes more current in the case of the appearance of redundancy when the robot has more degrees of freedom than would be necessary and sufficient to ensure the position and orientation imposed on the operational level. A way of treating problems in this case makes the use of the differential model of the robot and its pseudo inverse matrix. The numerical simulations in this case involve a large number of calculations, which is why the use of programs that include packages of symbolic calculation functions is indicated. In this paper it was analyzed the case of a robot with five degrees of freedom whose task was to ensure a trajectory imposed at the origin of the coordinate system attached to its end effector. The redundancy was analyzed by imposing as an optimization criterion the achievement of a minimum of displacements at the motor axes level of the analyzed robot. The results obtained with a program developed by the authors using Maple present the variation of motors coordinates in the case of a linear interpolation of the end effector movement. The obtained variation curves of the motors coordinates highlight a continuous variation of them, without presenting sudden variations that will lead to improper operation of the robot. It is also mentioned that the methodology generating the inverse geometric model in the redundancy case studied and the simulator developed by the authors offer the possibility of analyzing also other optimization criteria than the one presented.

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